Lab 1: Measuring the Real World

Mikel F.Qafa

September 9, 2012

Abstract

There is a reason why we have the words 'error' and 'estimate' in our math vocabulary, because nothing in the real world is as perfect as displayed on paper. These two act both as quantitative and qualitative values for our experiments. We would probably never find the exact same weather conditions two days in a raw because nature is dependent on many variables. In the same way, we cannot be perfectly sure that we have measured the exact correct length of something. With this lab I have tried to determine the area of an enclosed area on campus, shaped like a rectangle, and for this I had to use the length of the sides of this rectangle. Measuring the lengths and calculating the area proved to be more challenging that I had thought because none of the sides were the same length (according to my measurements). I have tried to show the assumptions I have made so that it makes it easier to understand why the results differ from each other. I have used 5 different tools to take the measurements, Google Earth, two measuring wheels, a measuring tape and my footsteps. It is very important to note that I have assumed the earth is flat and that some of the angles of the rectangle are right angles, even though the only tool I used to determine that were my eyes. I have taken three measurements of each side with each different tool. I have rounded all the measurements up to two decimal figures. Using the surface area obtained from the calculations I have tried to estimate the absolute and relative error for each of the different areas.

Determining the area with Google Earth.

Figure 1 shows where I have placed the placemarks on the Google Earth window. On the first run I used two points as reference points, which means that I assumed they are the correct corners of the rectangle, and then using the coordinates of these two reference points I was able to place the other two placemarks, and as expected they formed a perfect rectangle. The length was measured using the program's "Direction" tool, which tells you the distance between two points as a walking distance. For the second set of data I used the other two corners as reference points, and in the third run I used the coordinates of each point as they were on reference points. The resolution of the view is not very impressive, so we cannot determine whether it is on the accurate points, I took point P1 and P4 , as represented on the picture below. After determining the most accurate and precise points P1 and P4 , I found point P2 and P3. The coordinates of the reference points are displayed in Table 1.

Figure 1

Using the assumption that the area is a rectangle, I determine that points P1 & P2, P3 & P4 should have the same longitude. For the same reason, points P2 & P4 and P1 & P3 have the same latitude. The coordinates of all the points are displayed in Table 1.

Point	Longitude	Latitude
P1	84º54’46.36” W	39º49’25.85” N
P2	84º54’46.36” W	39º49’24.35” N
P3	84º54’43.78” W	39º49’25.85” N
P4	84º54’43.78” W	39º49’24.35” N

Table 1

The next step was to take two other reference points, P2 and P3 now. In Table 2 I have listed the data gathered,

Point	Longitude	Latitude
P1	84º54’46.38” W	39º49’25.82” N
P2	84º54’46.38” W	39º49’24.33” N
P3	84º54’43.74” W	39º49’25.82” N
P4	84º54’43.74” W	39º49’24.33” N

Table 2

Again, as in the first time, the coordinates of the two reference points dictate the coordinates of the the other two reference points.

In do not need to do the third measurements because now I already have the data, because I used the reference coordinates for each of the points.

The results are shown in Table 3.

Point	Longitude	Latitude
P1	84º54'46.36" W	39º49'25.85" N
P2	84º54'46.38" W	39º49'24.33" N
P3	84º54'43.74" W	39º49'25.82" N
P4	84º54'43.78" W	39º49'24.35" N

Table 3

Then it was time to determine the distance between the points for each case.

The length of the sides of the rectangle I got using Google Earth were the same for each case and they are displayed in Table 4.

Point-to-point	Distance in feet	Distance in meters
P1 to P2	154	46.94
P2 to P4	207	63.09
P3 to P4	154	46.94
P1 to P3	207	63.09

Table 4

The results have been rounded up to two decimal figures in meters, but Google earth gave the distance in feet without decimal figures.

Now I have all the data needed for the Google Earth method. I have three measurements that are the same and if I average them I could get the average distances of both sides of the rectangle, and then I could find the area of the rectangle. There is no need to calculate the average in this case because the numbers are

the same in all three cases. The formula for calculating the area of a rectangle is width * height, and we know both of these variables, so we just plug them in

the equation:

Width * height = 63.09 * 46.94 = 2961.44 m^2.

That is the area of the surface using Google Earth as a tool. Maybe it is not such a good tool in terms of accuracy or precision, but it is a very quick tool

in determining an approximation of the surface area.

BIG WHEEL

One of the other tools I used to measure the distance was the large measuring wheel. It is very easy to use and quick as well.

Figure 2

Figure 2 shows a picture of the tool used for this part of the experiment. During the measurements I found that the enclosed area was not a perfect rectangle at all,

so in order to make it easier for the calculations, and avoid more error, I divided the "rectangle" in two triangles which do not have the same diagonal length,

and therefore their sides are of different length too. I have displayed that in Figure 3,

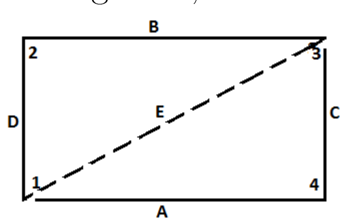

Figure 3

Triangle AEC is smaller than BED, where E is the diagonal, and it is different for both triangles. I have assumed that angles in the corners 4 and 2 are right angles.

In table 5 are shown that gathered data and the calculations. The measurements were made in feet because of the tool, but I converted the length in meters and

then found the average of those three measurements for each different section.

Section	Length in feet	Length in meters	Avg length (m)
A	202.8	61.81	61.81
	202.7	61.78	
	202.9	61.84	
B	202.2	61.63	61.67
	202.5	61.72	
	202.3	61.66	
C	147.3	44.90	44.93
	147.5	44.96	
	147.4	44.93	
D	150.8	45.96	45.95
	150.7	45.94	
	150.7	45.94	

Table 5

Since we are using the two right triangles, we need to know the length of the diagonal, which can be found using data from table 5 and Pythagorean theorem.

I have used the average of the lengths to find the respective diagonals.:

Triangle	Diagonal length E (m)	Area (m^2)
AEC	$\sqrt{A^2+C^2}=76.41$	$\frac{1}{2}A*C=1388.56$
BED	$\sqrt{B^2+D^2}=76.91$	$\frac{1}{2}B*D=1416.87$

Table 6

Now, the total area of the rectangle is the sum of the areas of these two triangles, AEC and BED: $1388.56+1416.87=2805.43$ m^2 .

SMALL WHEEL

The other tool I used, the small measuring wheel, is shown in Figure 4.

Figure 4

The same assumptions made in the case of the bigger wheel are made in this case as well. The data are processed in the same way as in Part II.

Table 7 shows all the measurements done with the small wheel.

Section	Length in feet	Length in meters	Avg length (m)
A	203.1	61.90	61.89
	202.9	61.84	
	203.2	61.93	
B	203.3	61.97	61.96
	203.4	61.99	
	203.2	61.93	
C	147.5	44.96	45.00
	147.8	45.05	
	147.6	44.99	
D	151.2	46.08	46.13
	151.5	46.18	
	151.3	46.12	

Table 7

In the same way as n Part II, we find the diagonals of the triangles and then the area of each triangle, and then add them together.

Triangle	Diagonal length (m)	Area (m^2)
AEC	$\sqrt{A^2+C^2} = 76.52$	$\frac{1}{2}A*C = 1392.52$
BED	$\sqrt{B^2+D^2} = 77.25$	$\frac{1}{2}B*D = 1429.11$

Table 8

Now, the total area of the rectangle is the sum of the areas of these two triangles, AEC and BED: $1392.52 + 1429.11 = 2821.63\ m^2$.

MEASURING TAPE

And now back to the more common measuring tools. The measuring tape was a very good tool because it was easier to use, and there was less room for error,

beside the problem that it might get stretched too much sometimes, and therefore influence the results.

Figure 5

The same procedure as for the other two tools followed for this one as well. Table 9 displays the data gathered using the measuring tape.

Section	Length in feet	Length in meters	Avg length (m)
A	202.6	61.75	61.74
	202.5	61.72	
	202.6	61.75	
B	202.2	61.63	61.62
	202.2	61.63	
	202.1	61.60	
C	147.3	44.88	44.85
	147.0	44.80	
	147.2	44.86	
D	150.7	45.93	45.91
	150.5	45.87	
	150.7	45.93	

Table 9

The same procedure follows in this case again, find the diagonals of the two triangles using the Pythagorean theorem for the right angles and then

calculated the area of each triangle. The diagonal is not actually used in any of the equations, but I have shown it so it would make it easier to distinguish the size of the triangle and to see how different these diagonals are, even though they come from the same 'rectangle'.

Triangle	Diagonal length (m)	Area (m^2)
AEC	$\sqrt{A^2 + C^2} = 76.31$	$\frac{1}{2}A * C = 1384.52$
BED	$\sqrt{B^2 + D^2} = 76.84$	$\frac{1}{2}B * D = 1414.49$

Table 10

Adding these areas together we get the total area of the surface measured using the measuring tape, AEC and BED: $1384.52 + 1414.49 = 2799.01$ m^2 .

Now we are left only with the no-technology-at-all method.

MY FOOTSTEPS

This method was done last and took longer than any other method, and I feel like it is the most inaccurate one for many reasons. Its is very hard that I

put my feet perfectly close to each other and they are not perfectly straight all the time because I tend to move. I measured my shoe size afterward

and it was roughly 10 inches long. 10 inches = 0.83 ft . As with any other tool, I walked accross the 4 sides of the rectangle 3 times, writing down

the number of shoe-size units of the distance. Data are shown in Table 11.

Section	Number of steps	Length in feet	Length in meters	Avg length (m)
A	245	203.35	61.98	61.81
	244	202.52	61.73	
	244	202.52	61.73	
B	240	199.2	60.71	60.46
	238	197.54	60.21	
	239	198.37	60.46	
C	177	146.91	44.78	44.51
	175	145.25	44.23	
	176	146.08	44.52	
D	181	150.23	45.79	45.70
	180	149.4	45.53	
	181	150.23	45.79	

Tab

11

In the same way that I did with the other four tools, I will find the area of the triangles and the diagonals of the triangles.

Triangle	Diagonal length (m)	Area (m^2)
AEC	$\sqrt{A^2 + C^2} = 76.16$	$\frac{1}{2}A * C = 1375.58$
BED	$\sqrt{B^2 + D^2} = 75.78$	$\frac{1}{2}B * D = 1381.51$

Table 12

And now adding the two last areas will give us: 1375.58 + 1381.51 = 2757.09 m² .

In table 13 I have gathered all the data for the enclosed area I have gathered using different methods:

Method	Area (m²)
Google Earth	2961.44
Large Wheel	2805.43
Small Wheel	2821.63
Meas. tape	2799.01
Footsteps	2757.09

Table 13

Only from the results it is almost impossible to tell which method was the correct one, or closer to correct, but when we take into account the assumptions,

the way the procedures were held and everything else, we understand why we got different results even though we were measuring the same sections.

Now, I cannot end the experiment without my final answer, which is supposed to tell the approximate area of the surface we have been inspecting.

$$\frac{2961.44 + 2805.43 + 2821.63 + 2799.01 + 2757.09}{4} = \frac{14187.37}{5} m^2 = 2837.43 m^2 \quad (1)$$

The absolute error is calculated with this formula: $e = A$ - avg, where A is the area obtained using one of the methods explained above and avg is the average

area from function (1).

The relative error is found by diving absolute error with the average area. Table 14 gives the areas with the absolute and relative errors:

Method	Area (m^2)	Absolute error (m^2)	Relative error
Google Earth	2961.44	2961.44-2837.43=123.57	$\frac{123.57}{2837.43}$ = 0.04(*4%*)
Large Wheel	2805.43	2805.43-2837.43= -32	$\frac{32}{2837.43}$ = 0.011(*1.1%*)
Small Wheel	2821.63	2821.63-2837.43= -15.8	$\frac{15.8}{2837.43}$ = 0.005(*0.5%*)
Meas. tape	2799.01	2799.01-2837.43= -38.42	$\frac{38.42}{2837.43}$ = 0.014(*1.4%*)
Footsteps	2757.09	2757.09-2837.43= -80.34	$\frac{80.34}{2837.43}$ = 0.028(*2.8%*)

Table 14

As we see the error is large because we are making error by as much as 123 m^2 and the lowest error being 15.8 m^2. So, if we were to tell which of the

tools is more accurate than the rest, the data from Table 14 tell me that I should choose the small measuring wheel because it has a relative error of less

than 1% and has the lowest absolute error too, which means that it is the closest area to the total average area of the enclosed area.

There are different sources of error for such exercise, our assumptions lead to errors, the tools I worked with are not perfect either, the surface on which I

rotated the wheels is not completely flat, and there are places that the wheel stops running even though distance has been passed, and many other errors

I have described throughout the data processing. This is the first time I am using LATEX, so there are many details in terms of presentation that need

to be addressed, and hopefully with practice I will become much better at it, and next time you read my lab report it will be more pleasing to your eyes.

However, there is always room for improvement, and for as long as we keep trying to get closer to the

correct result, we keep approaching the correct value.